Introduction to Data Mining

Mining Data Streams-1

U Kang
Seoul National University

Outline

- ☐ **Overview**
- ☐ Sampling From Data Stream
- ☐ Queries Over Sliding Window

Data Streams

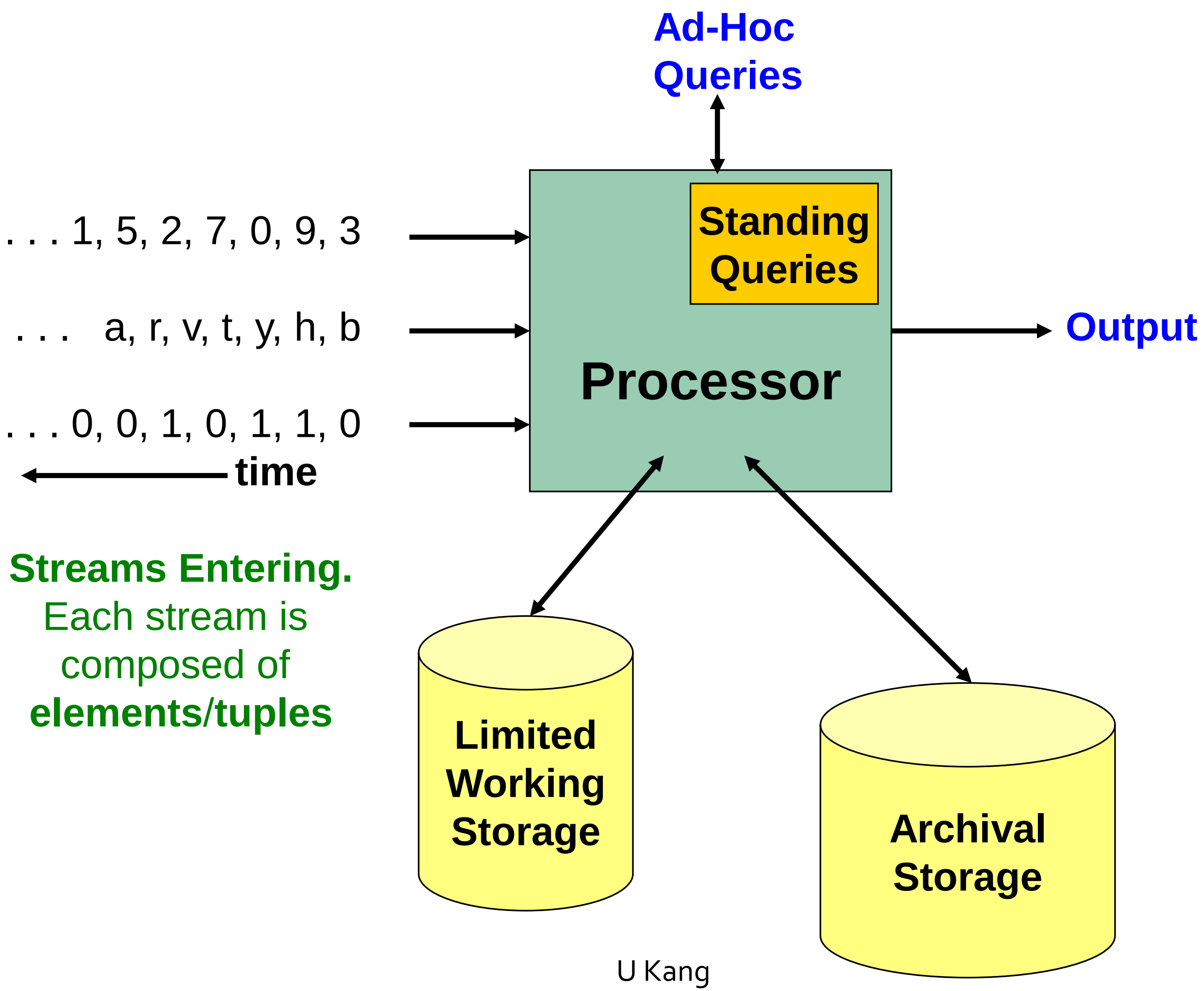
- **In many data mining situations, we do not know the entire data set in advance**
- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

The Stream Model

- Input **elements** enter at a rapid rate, at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
  - **The data do not fit in memory**
  - **Storing and accessing disks are slow**

- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

General Stream Processing Model



Problems on Data Streams

- **Types of queries one wants on answer on a data stream:**
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream

Problems on Data Streams

- **Types of queries one wants on answer on a data stream:**
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream
  - **Estimating moments**
    - Estimate avg./std. dev. of last ***k*** elements
  - **Finding frequent elements**

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday
- **Mining click streams**
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
  - Humidity, temperature, water leak, …
- **Telephone call records**
  - Data feed into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks

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- ☐ Queries Over Sliding Window

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as: How often did a user run the same query in a single day**
    - **= How often did a user run the same query at least twice?**
  - Have space to store **1/10th** of query stream

- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

- **Simple question: What fraction of queries by an average search engine user are duplicates?**
  - Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** queries)
    - **Correct answer: *d*/(*x*+*d*)**
  - **Proposed solution: We keep 10% of the queries**
    - Sample will contain ***x*/10** of the singleton queries and **d*19/100** of the duplicate queries at least once
    - But only ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - Of ***d*** "duplicates" ***18d/100*** appear exactly once
      - **18d/100 = ((1/10 · 9/10)+(9/10 · 1/10)) · d**
  - **So the sample-based answer is** $\frac{\frac{d}{100}}{\frac{x}{10}+\frac{d}{100}+\frac{18d}{100}} = \frac{d}{10x+19d}$

Solution: Sample Users

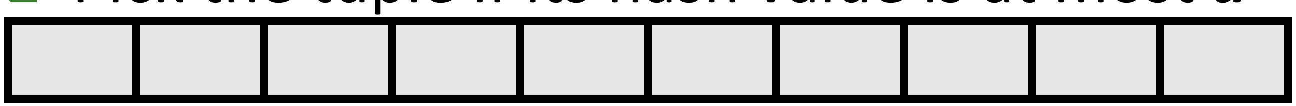
Solution:

- Pick **1/10th** of **users** and take all their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application
- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



E.g., How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose at time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

Very Clever!

- Algorithm (a.k.a. Reservoir Sampling)
  - Store all the first s elements of the stream to S
  - Suppose we have seen n-1 elements, and now the n^{th} element arrives ($n > s$)
    - With probability s/n, keep the n^{th} element, else discard it
    - If we picked the n^{th} element, then it replaces one of the s elements in the sample S, picked uniformly at random

- **Claim:** This algorithm maintains a sample S with the desired property:
  - After n elements, the sample contains each element seen so far with probability s/n

Proof: By Induction

- **We prove this by induction:**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing the ***n+1*** th element the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now *n+1* th element arrives**
- **Inductive step:** For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1***, tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1*** $= \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

Discussion

- Claim: At any time t, all the elements are given an equal probability s/t to be included in the samples
- Is the above claim true for the item that belongs to the samples at time n, and survived at time n+1? Yes (previous slide)
- Is the above claim true for the (n+1)th item?
  - The (n+1)th item may be discarded or included

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- ☐ **Queries Over Sliding Window**

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received

- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want to answer queries, how many times have we sold **X** in the last **k** sales

Sliding Window: 1 Stream

- **Sliding window on a single stream:**

N = 6

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past   Future →

Counting Bits (1)

- **Problem:**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form **How many 1s are in the last *k* bits?** where $k \leq N$

- **Obvious solution:**
  Store the most recent ***N*** bits
  - When new bit comes in, discard the ***N*+1**st bit

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2)

- **You can not get an exact answer without storing the entire window**

- **Real Problem:**
  What if we cannot afford to store *N* bits?
  - **E.g.**, we're processing 1 billion streams and ***N* = 1 billion**

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 ~~1 1 0 1 1 0~~

← Past Future →

- **But we are happy with an approximate answer**

An attempt: Simple solution

- **Q: How many 1s are in the last *N* bits?**
- A simple solution that does not really solve our problem: **Uniformity assumption**

N

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 0 1 1 0 0 1 1 0 1 0

Past Future

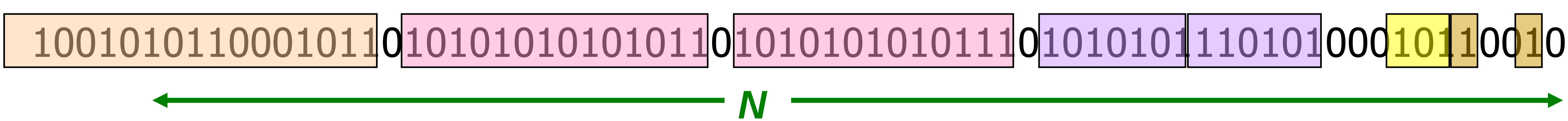
- **Maintain 2 counters:**
  - ***S***: number of 1s from the beginning of the stream
  - ***Z***: number of 0s from the beginning of the stream
- **How many 1s are in the last N bits?** $N \cdot \frac{S}{S+Z}$
- **But, what if stream is non-uniform?**
  - **What if distribution changes over time?**

DGIM Method

- **DGIM solution that does <u>not</u> assume uniformity**
- We store $\boldsymbol{O}(\log^2 \boldsymbol{N})$ bits per stream
- **Solution gives approximate answer, never off by more than 50%**
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits

DGIM method

- **Main Idea:** summarize blocks with specific number of **1s**, where the block ***sizes*** (number of **1s**) increase exponentially



DGIM: Timestamps

- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …
- Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in $\boldsymbol{O(log_2 N)}$ bits

DGIM: Buckets

- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its end** [O(log *N*) bits]
  - **(B) The number of 1s between its beginning and end** [O(log log *N*) bits]
    - **Proof: see below**
- **Constraint on buckets:**
  Number of **1s** must be a power of **2**
  - That explains the **O(log log *N*)** in **(B) above**

1001010110001011 0 1010101010101011 0 1010101010101111 0 1010101 110101 000 101 1 00 1 0

← *N* →

Representing a Stream by Buckets

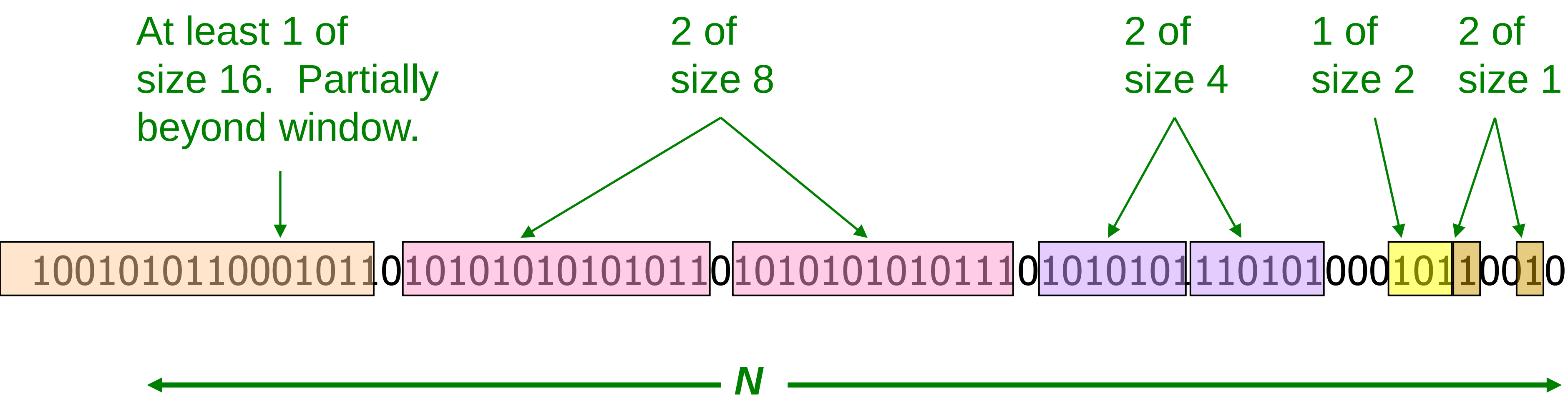
- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**

- **Buckets do not overlap in timestamps**

- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets

- Buckets disappear when their end-time is **>** ***N*** time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Storage Requirement

- The total number of buckets is O(log N). (why?)

- Each bucket requires O(log N) bits
  - **(A) The timestamp of its end** **[O(log *N*) bits]**
  - **(B) The number of 1s between its beginning and end** **[O(log log *N*) bits]**

- Thus, the total storage requirement is
  $O(\log N) * O(\log N) = O(\log^2 N)$

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time

- **2 cases:** Current bit is **0** or **1**

- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

- **If the current bit is 1:**
  - **(1)** Create a new bucket of size **1**, for just this bit
    - **End timestamp = current time**
  - **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
  - **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
  - **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

1001010110001011 0 1010101010101011 0 10101010101110 0 1010101 110101 000 101 1 00 1 0

Bit of value 1 arrives

0010101100010110 1010101010101011 0 10101010101110 0 1010101 110101 000 101 1 00 1 0 1

Two orange buckets get merged into a yellow bucket

0010101100010110 1010101010101011 0 10101010101110 0 1010101 110101 000 101 1001 0 1

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

0101100010110 1010101010101011 0 10101010101110 0 1010101 110101 000 101 1001 0 1 1 0 1

Buckets get merged…

0101100010110 1010101010101011 0 10101010101110 0 1010101 110101 000 101 1001 0 11 0 1

State of the buckets after merging

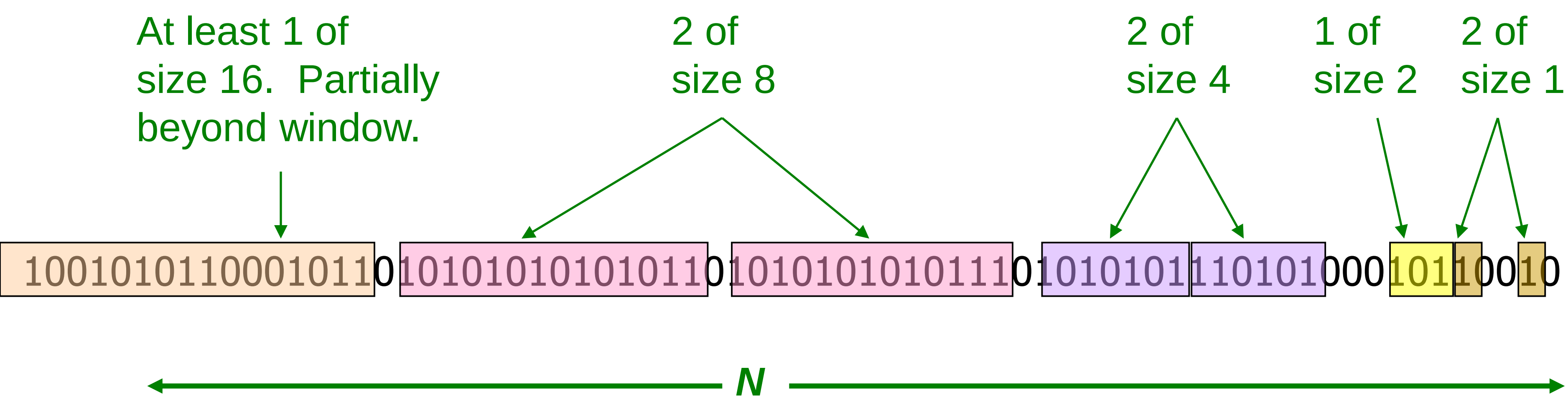
0101100010110 101010101010101110101010101110 0 1010101110101 000 1011001 0 11 0 1

How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets but the last**
     (note "size" means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

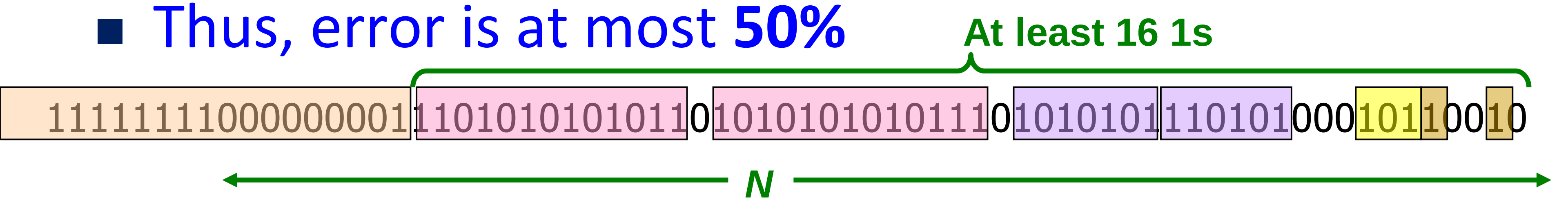
- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

Example: Bucketized Stream



Error Bound: Proof

- **Why is error 50%? Let's prove it!**
- Suppose the last bucket has size $\mathbf{2}^{r}$
- Then by assuming $\mathbf{2}^{r\text{-}1}$ (i.e., half) of its **1s** are still within the window, we make an error of at most $\mathbf{2}^{r\text{-}1}$
- Since there is at least one bucket of each of the sizes less than $\mathbf{2}^{r}$, the true sum is at least $\mathbf{1 + 2 + 4 + .. + 2^{r-1} = 2^{r} - 1}$
- Thus, error is at most **50%**



Further Reducing the Error

- Instead of maintaining **1** or **2** of each size bucket, we allow either ***x*-1** or ***x*** buckets (***x* > 2**)
  - Except for the largest size buckets; we can have any number between **1** and ***x*** of those
- **Error is at most *O(1/x)***
- By picking ***x*** appropriately, we can tradeoff between number of bits we store and the error
  - Increasing x => more memory space, less error

Extensions

- Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k* < *N***?
  - **A:** Find earliest bucket **B** that overlaps with ***k***. Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**

1001010110001011 0 1010101010101011 0 1010101010101111 0 1010101 110101 000 101 1 00 1 0

k

- **Can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?**

Extensions

- **Stream of positive integers**
- **We want the sum of the last *k* elements**
  - **Amazon:** Avg. price of last **k** sales
- **Solution:**
  - **If you know all integers have at most *m* bits**
    - Treat ***m*** bits of each integer as a separate stream
    - Use DGIM to count **1s** in each integer
    - The sum is $= \sum_{i=0}^{m-1} c_i 2^i$

$\boldsymbol{c_i}$ …estimated count for **i-th** bit

What You Need to Know

- **Sampling a fixed proportion of a stream**
  - Sample size grows as the stream grows
- **Sampling a fixed-size sample**
  - Reservoir sampling
- **Counting the number of 1s in the last N elements**
  - Exponentially increasing windows
  - Extensions:
    - Number of 1s in any last k (k < N) elements
    - Sums of integers in the last N elements

Questions?